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$$\begin{aligned}
 \therefore \Sigma \frac{1}{\sqrt{(R_a R_b)}} &= \frac{1}{r} \Sigma \sqrt{\frac{(1 - \sin \frac{1}{2} A)(1 - \sin \frac{1}{2} B)}{(1 + \sin \frac{1}{2} A)(1 + \sin \frac{1}{2} B)}} \\
 &= \frac{1}{r} \frac{\Sigma (\cos \frac{1}{4} A - \sin \frac{1}{4} A)(\cos \frac{1}{4} B - \sin \frac{1}{4} B)(\cos \frac{1}{4} C + \sin \frac{1}{4} C)}{\pi (\cos \frac{1}{4} A + \sin \frac{1}{4} A)(\cos \frac{1}{4} B + \sin \frac{1}{4} B)(\cos \frac{1}{4} C + \sin \frac{1}{4} C)} \\
 &= \frac{1}{r} \frac{\Sigma \cos \frac{A+\pi}{4} \cos \frac{B+\pi}{4} \cos \frac{C-\pi}{4}}{\pi \cos \frac{A-\pi}{4}} \\
 &= \frac{1}{r} \frac{\Sigma (\cos \frac{1}{2} A + \cos \frac{1}{2} B - \cos \frac{1}{2} C)}{\Sigma \cos \frac{1}{2} A} = \frac{1}{r}.
 \end{aligned}$$

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.

162. Proposed by *J. D. PALMER*, Providence, Ky.

Given the distances from the vertices of a triangle,  $ABC$ , to the center of the circle, to construct the triangle.

Solution by *G. B. M. ZERR*, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

$AO, BO, CO = a, b, c$ , respectively, where  $O$  is the center of the in-circle;  $BC, AC, AB = x, y, z$ , respectively. Let  $O_1$  be the center of the ex-circle opposite  $A$ . Then

$$AO^2 = \frac{(p-x)^2}{\cos^2 \frac{1}{2} A}, \text{ where } p = \frac{1}{2}(x+y+z).$$

$$\therefore AO^2 = \left[ \frac{p-x}{p} \right] yz = yz - \frac{xyz}{p} = yz - 4Rr = a^2.$$

$$\text{Similarly, } BO^2 = \left[ \frac{p-y}{p} \right] xz = xz - 4Rr = b^2.$$

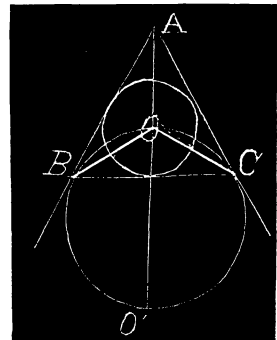
$$CO^2 = \left[ \frac{p-z}{p} \right] xy = xy - 4Rr = c^2.$$

$$\therefore AO \cdot BO \cdot CO = \frac{\Delta xyz}{p^2} = 4R^2 r^2 \text{ or } 2\sqrt{(abc)} = 4Rr$$

$$\therefore a^2 + 2\sqrt{(abc)} = yz, \quad b^2 + 2\sqrt{(abc)} = xz,$$

$$c^2 + 2\sqrt{(abc)} = xy.$$

$$\therefore x[a^2 + 2\sqrt{(abc)}] = y[b^2 + 2\sqrt{(abc)}] = z[c^2 + 2\sqrt{(abc)}]$$



$$= \sqrt{\{[a^2 + 2\sqrt{abc}][b^2 + 2\sqrt{abc}][c^2 + 2\sqrt{abc}]\}}.$$

This gives us the values of the sides.

Otherwise draw  $AO$  and produce  $AO$  to  $O_1$  so that  $OO_1 = 2\sqrt{bc/a}$ . Upon  $OO_1$  as diameter describe a circle. With  $O$  as a center and  $b$  as a radius describe an arc cutting the circle in  $B$ . Similarly, with  $O$  as center and  $c$  as radius, draw an arc cutting the circle in  $C$ . Join  $BC$ ,  $AC$ ,  $AB$ , then  $ABC$  is the triangle required. For  $O_1$  is the ex-center opposite  $A$  by construction as follows :

$$AO_1 = p/\cos \frac{1}{2}A. \quad \therefore AOA O_1 = yz = AO^2 + 2\sqrt{(AO \cdot BO \cdot CO)}.$$

$$\therefore AO_1 = AO + 2\sqrt{(BO \cdot CO)/AO}.$$

### CALCULUS.

121. Proposed by W. W. LANDIS, A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.

$$\text{Solve the differential equation } \left[ \frac{d}{dx} + b \right]^n y = \cos ax.$$

Solution by LON C. WALKER, A. M., Petaluma High School, Petaluma, Cal., and LEWIS NEIKIRK, B. S., Boulder, Col.

$$\left[ \frac{d}{dx} + b \right]^n y = \cos ax.$$

$$\left[ \frac{d}{dx} + b \right]^n \text{ has } n \text{ roots each } = -b.$$

$$\therefore \text{Comp. Factor} = e^{-bx}(c_1 + c_2x + c_3x^2 + c_4x^3 + \dots c_nx^{n-1}).$$

$$\frac{1}{\left[ \frac{d}{dx} + b \right]^n} \cos ax = \frac{\left[ \frac{d}{dx} - b \right]^n}{\left[ \frac{d}{dx} - b^2 \right]^n} \cos ax = \left\{ \frac{b - \frac{d}{dx}}{a^2 + b^2} \right\}^n \cos ax.$$

$$\text{Let } n=1. \quad \therefore \frac{b - d/dx}{a^2 + b^2} \cos ax = \frac{1}{a^2 + b^2} (b \cos ax + a \sin ax)$$

$$= \frac{1}{(a^2 + b^2)^{\frac{1}{2}}} \left[ \frac{b \cos ax + a \sin ax}{\sqrt{a^2 + b^2}} \right] \dots (1).$$

$$\text{Put } \theta = \cot^{-1} b/a, \text{ then } \sin \theta = \frac{a}{\sqrt{a^2 + b^2}}, \text{ and } \cos \theta = \frac{b}{\sqrt{a^2 + b^2}}.$$

$\therefore (1)$  reduces to

$$\frac{b - d/dx}{a^2 + b^2} \cos ax = (a^2 + b^2)^{-\frac{1}{2}} (\cos \theta \cos ax + \sin \theta \sin ax) = (a^2 + b^2)^{-\frac{1}{2}} \cos(ax - \theta).$$